B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

PRE BOARD-3, (2025-26)
MATHEMATICS (041) ,Set-1
Marking Key

Class: XII	**Time: 3hrs**
Date: 06/01/26	**Max Marks: 80**
Admission no:	**Roll no:**

General Instructions:

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1. **This Question Paper has 5 Sections A, B, C, D and E.**
2. **Section A has 20 MCQs carrying 1 mark each**
3. **Section B has 5 questions carrying 02 marks each.**
4. **Section C has 6 questions carrying 03 marks each.**
5. **Section D has 4 questions carrying 05 marks each.**
6. **Section E has 3 case-based integrated units of assessment (04 marks each) with sub-parts.**
7. **All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks have been provided. An internal choice has been provided in the 2marks questions of Section E**
8. **Draw neat figures wherever required. Take π =22/7 wherever required if not stated.**

	Section–A **This section comprises of MCQs of 1 mark each**	
Q.1	The principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is-----------------------. A. $\frac{-\pi}{3}$ B. $\frac{-2\pi}{3}$ C. $\frac{\pi}{3}$ **D.** $\frac{2\pi}{3}$	(1)
Q.2	If A = $[a_{ij}]$ is a 3 x 3 diagonal matrix such that a_{11} = 1, a_{22} = 5 and a_{33}= -2 then $\|A\|$ is-------. A.0 **B. -10** C. 10 D. 1	(1)
Q.3	The area of the region bounded by the curve $x^2 = y$ and the line y = 4 is---------. A. 32 sq units **B. $\frac{32}{3}$ sq units** C. $\frac{1}{32}$ sq units D. $\frac{1}{3}$ sq units	(1)
Q.4	The function f(x) = $4\sin^3x - 6\sin^2x + 12\sin x + 100$ is strictly--------------------------. A. increasing in $\left(\pi, \frac{3\pi}{2}\right)$ **B. decreasing in** $\left(\frac{\pi}{2}, \pi\right)$ C. decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ D. decreasing in $\left[0, \frac{\pi}{2}\right]$	(1)
Q.5	If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$ then the value of (x + y) is--------------. A. 1 B. 2 C. 4 **D. 5**	(1)
Q.6	The value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ is-----------------. **A.** $\frac{\pi}{3}$ B. $\frac{2\pi}{3}$ C. $\frac{-\pi}{3}$ D. $\frac{\pi}{6}$	(1)
Q.7	The value of the determinant $\begin{vmatrix} x-1 & 1 \\ x^3 & x^2+x+1 \end{vmatrix}$ is-----------------. A. 3 B. 0 **C. -1** D. 1	(1)
Q.8	If y = x^3 + tanx then y'' – 2 $\sec^2x$ tanx is ----------------------------. A. 6 **B. 6x** C. 3 D. 3x	(1)
Q.9	All the points of discontinuity of f defined by f(x) = $\|x\|$ - $\|x+1\|$ is/are------------. A. 0,1 B. 1, 0, 2 **C. No point of discontinuity** D. None of the above	(1)
Q.10	For matrix A = $\begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$, (A')A is equal to--------------------. **A.** $\begin{bmatrix} 10 & 13 \\ 13 & 29 \end{bmatrix}$ B. $\begin{bmatrix} 10 & 13 \\ 29 & 13 \end{bmatrix}$ C. $\begin{bmatrix} 13 & 29 \\ 10 & 13 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 10 \\ 1 & 10 \end{bmatrix}$	(1)

Q.11	$\int \frac{dx}{\sqrt{x}+x}$ is equal to------------------------. **A. $2\log\left\|\sqrt{x}+1\right\|$ + c** B. $\log\|x+1\|$ + c C. $\log\|x-1\|$ + c D. $2\log\|x+1\|$ + c	(1)
Q.12	Degree of the differential equation $\frac{d^2y}{dx^2} + e^{\frac{dy}{dx}} = 0$. A. 1 B. 2 C. 3 **D. Not defined**	(1)
Q.13	The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2$ is-------------------. A. x **B. $\frac{1}{x}$** C. $x^{\frac{1}{2}}$ D. $x^{\frac{3}{2}}$	(1)
Q.14	If $\vec{a} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and $\vec{b} = 6\hat{i} - \hat{j} - 5\hat{k}$, then $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$ is-----------------. **A. 24** B. -24 C. 18 D. 10	(1)
Q.15	The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is------------. A. $\frac{5\pi}{6}$ **B. $\frac{3\pi}{4}$** C. $\frac{5\pi}{4}$ D. $\frac{7\pi}{4}$	(1)
Q.16	The probability that A speaks truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is-----------------. **A. $\frac{7}{20}$** B. $\frac{1}{5}$ C. $\frac{3}{20}$ D. $\frac{4}{5}$	(1)
Q.17	In LPP, if the objective function Z = ax + by has the same maximum value on two corner points of the feasible region, then the number of points at which Z_{max} occurs is----------------. A. 0 B. 2 C. finite **D. infinite**	(1)
Q.18	The angle between the lines 2x = 3y = -z and 6x = -y = -4z is-------------------. A. 0° B. 30° C. 45° **D. 90°**	(1)
	Followings are Assertion-Reason based questions in which a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. A. Both A and R are true and R is the correct explanation of A. B. Both A and R are true and R is not the correct explanation of A. C. A is true but R is false. D. A is false but R is true.	
Q.19	Assertion (A) : The function f: R → R given by f(x) = [x] + x is one-one and onto. Reason (R) : A function is said to be one –one and onto, if each element has unique image and range of f(x) is equal to codomain of f(x). D	(1)
Q.20	Assertion (A) : If $\|2\vec{a} + \vec{b}\| = \|2\vec{a} - \vec{b}\|$, then $\vec{a}$ parallel to $\vec{b}$. Reason (R) : Two non-zero vectors $\vec{a}$ and $\vec{b}$ are perpendicular, if $\vec{a} \cdot \vec{b} = 0$. D	(1)
	Section–B **This section comprises of very short answer type questions of 2 marks each**	
Q.21 Sol:	Find the domain of the function f(x) = $\sin^{-1}(2x-5)$. $-1 \le 2x - 5 \le 1$ 3. **Solve the inequality for x:** To isolate x, add 5 to all parts of the compound inequality: $-1 + 5 \le 2x - 5 + 5 \le 1 + 5$ $4 \le 2x \le 6$ 4. **Divide all parts by 2:** $\frac{4}{2} \le \frac{2x}{2} \le \frac{6}{2}$ x∈ (2,3)	(2)
Q.22 Sol:	Examine the continuity of the function f(x) = $\begin{cases} \frac{x}{2\|x\|}, & if\ x \ne 0 \\ \frac{1}{2}, & if\ x = 0 \end{cases}$ at x = 0.	(2)

	1. **Left-hand limit (LHL):** As x approaches 0 from the left ($x < 0$), $\|x\| = -x$. $$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} \frac{x}{2\|x\|} = \lim_{x\to0^-} \frac{x}{2(-x)} = \lim_{x\to0^-} \frac{x}{-2x} = \lim_{x\to0^-} -\frac{1}{2} = -\frac{1}{2}$$ 2. **Right-hand limit (RHL):** As x approaches 0 from the right ($x > 0$), $\|x\| = x$. $$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{x}{2\|x\|} = \lim_{x\to0^+} \frac{x}{2(x)} = \lim_{x\to0^+} \frac{x}{2x} = \lim_{x\to0^+} \frac{1}{2} = \frac{1}{2}$$ Point of discontinuity is 0.	
Q.23 Sol:	The sum of two unit vectors is a unit vector, then show that the magnitude of their difference is $\sqrt{3}$. The magnitude squared of the sum is $\|\mathbf{a}+\mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\mathbf{a}\cdot\mathbf{b}$. Substituting the known values and using the dot product formula $\mathbf{a}\cdot\mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$: $$1^2 = 1^2 + 1^2 + 2(1)(1)\cos\theta$$ $$1 = 2 + 2\cos\theta$$ Solving for $\cos\theta$ yields: $$\cos\theta = -\frac{1}{2}$$ The magnitude squared of the difference is $\|\mathbf{a}-\mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\mathbf{a}\cdot\mathbf{b}$. Substituting the magnitudes and the value of $\cos\theta$: $$\|\mathbf{a}-\mathbf{b}\|^2 = 1^2 + 1^2 - 2(1)(1)\left(-\frac{1}{2}\right)$$ $$\|\mathbf{a}-\mathbf{b}\|^2 = 1 + 1 + 1$$ $$\|\mathbf{a}-\mathbf{b}\|^2 = 3$$ Taking the square root gives the magnitude of the difference: $$\|\mathbf{a}-\mathbf{b}\| = \sqrt{3}$$	(2)
Q.24 Sol:	Evaluate $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$ dx. **Substitution:** Let $t = e^x$. Then the differential $dt = e^x dx$. The integral can be rewritten in terms of t: $$\int \frac{dt}{\sqrt{5-4t-t^2}}$$	(2)

Rewrite the Integral: The integral now becomes:

$$\int \frac{dt}{\sqrt{3^2-(t+2)^2}}$$

Apply Standard Formula: This integral has the standard form

$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$. Here, $a = 3$ and $u = t+2$.

$$\int \frac{dt}{\sqrt{3^2-(t+2)^2}} = \sin^{-1}\left(\frac{t+2}{3}\right) + C$$

$\text{Sin}^{-1}\left(\frac{e^x+2}{3}\right)$ +C

(OR)

Find the value of $\int \frac{cosx}{(1+sinx)(2+sinx)}$ dx

We use the substitution $u = \sin(x)$. Differentiating with respect to x gives $du = \cos(x)\,dx$. The integral transforms into:

$$\int \frac{1}{(1+u)(2+u)}\,du$$

We decompose the integrand into partial fractions:

$$\frac{1}{(1+u)(2+u)} = \frac{A}{1+u} + \frac{B}{2+u}$$

Solving for A and B, we find $A = 1$ and $B = -1$. The expression becomes:

$$\frac{1}{1+u} - \frac{1}{2+u}$$

We integrate the decomposed expression with respect to u:

$$\int \left(\frac{1}{1+u} - \frac{1}{2+u}\right) du = \ln|1+u| - \ln|2+u| + C$$

Using logarithm properties, this simplifies to:

$$\ln\left|\frac{1+u}{2+u}\right| + C$$

$\log\left|\frac{1+sinx}{2+sinx}\right|$ +C

Q.25 Find the values of x, y and z, if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A' = A^{-1}. (2)

SoL: The condition $A' = A^{-1}$ means that the matrix A is an **orthogonal matrix**. This condition is equivalent to $A'A = I$ (or $AA' = I$), where I is the identity matrix.

	From the first row, first column: $0 \cdot 0 + x \cdot x + x \cdot x = 2x^2 = 1 \Longrightarrow x^2 = 1/2 \Longrightarrow x = \pm 1/\sqrt{2}$. From the second row, second column: $2y \cdot 2y + y \cdot y + (-y) \cdot (-y) = 4y^2 + y^2 + y^2 = 6y^2 = 1 \Longrightarrow y^2 = 1/6 \Longrightarrow y = \pm 1/\sqrt{6}$. From the third row, third column: $z \cdot z + (-z) \cdot (-z) + z \cdot z = z^2 + z^2 + z^2 = 3z^2 = 1 \Longrightarrow z^2 = 1/3 \Longrightarrow z = \pm 1/\sqrt{3}$ **x = ±1/√2** **y = ±1/√6** **z = ±1/√3**	
	Section–C **This section comprises of short answer type questions of 3 marks each**	
Q.26 Sol:	Show that the relation R on the set Z of all integers defined by (x, y) ∈ R ⇔(x – y) is divisible by 3 is an equivalence relation. Prove Reflexivity: Prove Symmetric Prove Transitivity Hence, Relation is Equivalence.	(3) 1 1 1
Q.27 Sol:	Show that A = $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is sum of symmetric and skew symmetric matrices. Find ½ (A+A') Find ½ (A-A') Add both We will get matrix A.	(3) 1 1 1
Q.28 Sol:	Find the derivative of $x^{\log x}$ w.r.t logx. Let $u = x^{\log x}$ (the function to differentiate). Let $v = \log x$ (the variable with respect to which we differentiate). Take the natural log of both sides: $\ln(u) = \ln(x^{\log x}) = (\log x)(\log x) = (\log x)^2$. Differentiate with respect to x (using chain rule): $\frac{1}{u}\frac{du}{dx} = 2(\log x) \cdot \frac{1}{x}$. Solve for $\frac{du}{dx}$: $\frac{du}{dx} = u \cdot \frac{2 \log x}{x} = x^{\log x} \cdot \frac{2 \log x}{x}$. Simplify: $\frac{du}{dx} = 2(\log x)x^{\log x - 1}$.	(3)

	Find $\frac{dv}{dx}$ (Derivative of v w.r.t. x): • $\frac{dv}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$. Apply the Chain Rule ($\frac{du}{dv} = \frac{du/dx}{dv/dx}$): • $\frac{du}{dv} = \frac{2(\log x)x^{\log x - 1}}{1/x}$. • $\frac{du}{dv} = 2(\log x)x^{\log x - 1} \cdot x$. • $\frac{du}{dv} = 2(\log x)x^{\log x - 1 + 1}$. • $\frac{du}{dv} = 2(\log x)x^{\log x}$. **(OR)** If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **Substitute variables**: Let $x = \sin\theta$ and $y = \sin\Phi$. This means $\theta = \sin^{-1} x$ and $\Phi = \sin^{-1} y$. The given equation becomes: $\sqrt{1-\sin^2\theta} + \sqrt{1-\sin^2\Phi} = a(\sin\theta - \sin\Phi)$ $\cos\theta + \cos\Phi = a(\sin\theta - \sin\Phi)$ **Apply trigonometric identities**: Use the sum-to-product identities: 1. $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ 2. $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ Substituting these into the equation: $2\cos\left(\frac{\theta+\Phi}{2}\right)\cos\left(\frac{\theta-\Phi}{2}\right) = 2a\cos\left(\frac{\theta+\Phi}{2}\right)\sin\left(\frac{\theta-\Phi}{2}\right)$ **Simplify the equation**: Divide both sides by $2\cos\left(\frac{\theta+\Phi}{2}\right)\sin\left(\frac{\theta-\Phi}{2}\right)$ (assuming it is non-zero): $\frac{\cos\left(\frac{\theta-\Phi}{2}\right)}{\sin\left(\frac{\theta-\Phi}{2}\right)} = a$ $\cot\left(\frac{\theta-\Phi}{2}\right) = a$ **Solve for $\theta - \Phi$**: Take the inverse cotangent (or arctan of $1/a$) of both sides: $\frac{\theta-\Phi}{2} = \cot^{-1}(a)$ $\theta - \Phi = 2\cot^{-1}(a)$ **Substitute back x and y**: Replace θ and Φ with their inverse sine expressions: $\sin^{-1} x - \sin^{-1} y = 2\cot^{-1}(a)$ Note that the right-hand side, $2\cot^{-1}(a)$, is a constant.	

	$$\frac{d}{dx}(\sin^{-1}x) - \frac{d}{dx}(\sin^{-1}y) = \frac{d}{dx}(2\cot^{-1}(a))$$ $$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$ (using the chain rule for the y term and noting the derivative of a constant is zero). **Isolate $\frac{dy}{dx}$:** $$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}$$ $$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$ **Final Result:** $$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$	
Q.29 Sol:	Sand pouring from a pipe at the rate of 15cm^3/min. The falling sand forms a cone on the ground such that the height of the cone is always one third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4cm? Substitute $r = 3h$ into the volume formula to express V solely as a function of h: $$V = \frac{1}{3}\pi(3h)^2 h$$ $$V = 3\pi h^3$$ $$\frac{dV}{dt} = \frac{d}{dt}(3\pi h^3)$$ $$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$$ Rearrange the equation to solve for $\frac{dh}{dt}$: $$\frac{dh}{dt} = \frac{dV/dt}{9\pi h^2}$$ Substitute the given values $\frac{dV}{dt} = 15\,\text{cm}^3/\text{min}$ and $h = 4\,\text{cm}$: $$\frac{dh}{dt} = \frac{15}{9\pi(4)^2}$$ $$\frac{dh}{dt} = \frac{15}{144\pi}$$ $$\frac{dh}{dt} = \frac{5}{48\pi}\ \text{cm/min}$$	(3)

	(OR) A spherical ball of salt is dissolving in water in such a manner that the rate of decreasing of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate. Using the chain rule on the volume formula: $\frac{dV}{dt} = \frac{d}{dt}(\frac{4}{3}\pi r^3) = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$. This simplifies to: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. Substitute the expressions for $\frac{dV}{dt}$ and S into the given proportional relationship: $4\pi r^2 \frac{dr}{dt} = -k(4\pi r^2)$. Cancel the common term $4\pi r^2$ from both sides: $\frac{dr}{dt} = -k$.	
Q.30 Sol:	If the unit vector $\vec{a}$ makes angle $\frac{\pi}{4}$ with $\hat{\imath}$, $\frac{\pi}{3}$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$, then find the components of $\vec{a}$ and the angle θ. The sum of the squares of the direction cosines for any vector is always unity, i.e., $\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$. Substituting the given angles: $$\cos^2(\pi/4) + \cos^2(\pi/3) + \cos^2(\theta) = 1$$ We know that $\cos(\pi/4) = 1/\sqrt{2}$ and $\cos(\pi/3) = 1/2$. $$(\frac{1}{\sqrt{2}})^2 + (\frac{1}{2})^2 + \cos^2(\theta) = 1$$ $$\frac{1}{2} + \frac{1}{4} + \cos^2(\theta) = 1$$ $$\frac{3}{4} + \cos^2(\theta) = 1$$ Solving for $\cos^2(\theta)$ gives $\cos^2(\theta) = 1 - \frac{3}{4} = \frac{1}{4}$. Thus, $\cos(\theta) = \pm\frac{1}{2}$. Since θ is an acute angle, we take the positive value, $\cos(\theta) = \frac{1}{2}$, which means $\theta = \frac{\pi}{3}$. The components of $\mathbf{a}$ are its direction cosines: $$a_x = \cos(\pi/4) = \frac{1}{\sqrt{2}}$$ $$a_y = \cos(\pi/3) = \frac{1}{2}$$ $$a_z = \cos(\theta) = \frac{1}{2}$$	(3)

Q.31 Sol:	Among the students in a college, it is known that 60% reside in hostel and 40% are day scholars. Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual exams. At the end of year, one student is chosen at random from the college and he has A grade, what is the probability that the student is a hosteller? $P(H) = 0.60$ $P(D) = 0.40$ $P(A \mid H) = 0.30$ $P(A \mid D) = 0.20$ To use Bayes' theorem, we first calculate the total probability of a student getting an A grade, $P(A)$, using the law of total probability: $$P(A) = P(A \mid H)P(H) + P(A \mid D)P(D)$$ $$P(A) = (0.30)(0.60) + (0.20)(0.40)$$ $$P(A) = 0.18 + 0.08 = 0.26$$ We want to find the probability that a student is a hosteller given they have an A grade, $P(H \mid A)$, using Bayes' theorem: $$P(H \mid A) = \frac{P(A \mid H)P(H)}{P(A)}$$ $$P(H \mid A) = \frac{0.18}{0.26} = \frac{18}{26} = \frac{9}{13}$$	(3)
	Section–D **This section comprises of long answer type questions of 5 marks each**	
Q.32	Find the coordinates of the foot of perpendicular drawn from the point (2, 3, -8) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the perpendicular distance of the given point from the line. The given equation of the line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. We can rewrite this in the standard symmetric form: $$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$$ Any general point $\boldsymbol{M}$ on this line can be expressed in terms of the parameter λ as $\boldsymbol{M}(4-2\lambda, 6\lambda, 1-3\lambda)$. The direction ratios of the line are proportional to $\mathbf{d} = \langle -2, 6, -3 \rangle$.	(5)

	Let $P(2,3,-8)$ be the given point. The direction ratios of the line segment PM are given by the difference in coordinates: $$\mathbf{PM} = \langle (4-2\lambda)-2, 6\lambda-3, (1-3\lambda)-(-8) \rangle$$ $$\mathbf{PM} = \langle 2-2\lambda, 6\lambda-3, 9-3\lambda \rangle$$ These direction ratios are proportional to $\langle 2-2\lambda, 6\lambda-3, 9-3\lambda \rangle$. Since PM is perpendicular to the given line, the dot product of their direction ratios must be zero: $$\mathbf{PM}\cdot\mathbf{d} = 0$$ $$(2-2\lambda)(-2)+(6\lambda-3)(6)+(9-3\lambda)(-3)=0$$ $$-4+4\lambda+36\lambda-18-27+9\lambda=0$$ $$49\lambda-49=0$$ $$\lambda=1$$ Substitute $\lambda=1$ into the coordinates of point M: $$M(4-2(1), 6(1), 1-3(1))$$ $$M(2,6,-2)$$ The coordinates of the foot of the perpendicular are **(2, 6, -2)**. The perpendicular distance is the distance between point $P(2,3,-8)$ and point $M(2,6,-2)$. We use the distance formula: $$PM = \sqrt{(2-2)^2+(6-3)^2+(-2-(-8))^2}$$ $$PM = \sqrt{0^2+3^2+6^2}$$ $$PM = \sqrt{0+9+36}$$ $$PM = \sqrt{45}$$ $$PM = 3\sqrt{5}$$ The perpendicular distance is **$3\sqrt{5}$ units.**	
Q.33	Solve the following LPP graphically. Maximise Z = 300x + 600y Subject to constraints, $x + 2y \le 12$, $2x + y \le 12$, $x + \frac{5}{4}y \ge 5$ and $x \ge 0$, $y \ge 0$.	(5)

	Graph the Constraint Equations: • For **x + 2y = 12**: Points are (0, 6) and (12, 0). • For **2x + y = 12**: Points are (0, 12) and (6, 0). • For **x + 5/4y = 5**: Points are (0, 4) and (5, 0). • The non-negativity constraints **x ≥ 0, y ≥ 0** restrict the region to the first quadrant. The corner points of the feasible region are found by identifying the intersectior points of the boundary lines: • Intersection of `x + 5/4y = 5` and the y-axis (x=0): **(0, 4)**. • Intersection of `x + 5/4y = 5` and the x-axis (y=0): **(5, 0)**. • Intersection of `2x + y = 12` and the x-axis (y=0): **(6, 0)**. • Intersection of `x + 2y = 12` and the y-axis (x=0): **(0, 6)**. • Intersection of `x + 2y = 12` and `2x + y = 12`: Solving these simultaneously gives **(4, 4)**. The actual corner points defining the *feasible* region are **(0, 4)**, **(5, 0)**, **(6, 0)**, **(4, 4)**, and **(0, 6)** (based on analysis of which points form the final boundary). **Evaluate the Objective Function at Each Corner Point:** • At **(0, 4)**: Z = 300(0) + 600(4) = **2400**. • At **(5, 0)**: Z = 300(5) + 600(0) = **1500**. • At **(6, 0)**: Z = 300(6) + 600(0) = **1800**. • At **(4, 4)**: Z = 300(4) + 600(4) = 1200 + 2400 = **3600**. • At **(0, 6)**: Z = 300(0) + 600(6) = **3600**. The maximum value at (0,6) is 3600.	
Q.34	Evaluate $\int_0^{\pi} x\, log\lvert sin\, x\rvert dx$. Let $I = \int_0^{\pi} x \log(\sin x)\, dx$. (The absolute value sign can be dropped since $\sin x \geq 0$ for $x \in [0, \pi]$). Apply the property $\int_0^{a} f(x)\, dx = \int_0^{a} f(a-x)\, dx$. Here, $a = \pi$. $I = \int_0^{\pi} (\pi - x) \log(\sin(\pi - x))\, dx$. $I = \int_0^{\pi} (\pi - x) \log(\sin x)\, dx$. $I = \int_0^{\pi} \pi \log(\sin x)\, dx - \int_0^{\pi} x \log(\sin x)\, dx$. $I = \pi \int_0^{\pi} \log(\sin x)\, dx - I$.	(5)

$I = \int_0^{\pi} (\pi - x) \log(\sin x)\, dx.$

$I = \int_0^{\pi} \pi \log(\sin x)\, dx - \int_0^{\pi} x \log(\sin x)\, dx.$

$I = \pi \int_0^{\pi} \log(\sin x)\, dx - I.$

$I = \frac{\pi}{2} (-\pi \log 2).$

$I = -\frac{\pi^2}{2} \log 2.$

(OR)

Evaluate $\int_0^{\frac{\pi}{2}} (2\log(sinx) - \log(sin2x))$dx.

We can rewrite the original integral into three separate integrals based on the simplified integrand:

$$I = \int_0^{\pi/2} (\log(\sin(x)) - \log(\cos(x)) - \log(2))dx$$

$$I = \int_0^{\pi/2} \log(\sin(x))dx - \int_0^{\pi/2} \log(\cos(x))dx - \int_0^{\pi/2} \log(2)dx$$

Let $J = \int_0^{\pi/2} \log(\sin(x))dx$ and $K = \int_0^{\pi/2} \log(\cos(x))dx$. Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we find

$K = \int_0^{\pi/2} \log(\cos(\pi/2 - x))dx = \int_0^{\pi/2} \log(\sin(x))dx = J.$

Thus, the first two terms cancel out:

$$J - K = J - J = 0$$

The third integral is straightforward:

$$\int_0^{\pi/2} \log(2)dx = \log(2)[x]_0^{\pi/2} = \log(2)\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{2}\log(2)$$

Combining these results:

$$I = 0 - \frac{\pi}{2}\log(2) = -\frac{\pi}{2}\log(2)$$

Q.35 Solve the differential equation xdy – ydx = $\sqrt{x^2 + y^2}$ dx. (5)

The given differential equation is $xdy - ydx = \sqrt{x^2 + y^2}\, dx$. Rearranging terms to express it in the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$:

$$xdy = (y + \sqrt{x^2 + y^2})dx$$

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} = \frac{y}{x} + \sqrt{\frac{x^2 + y^2}{x^2}} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}$$

This is a homogeneous equation. We use the substitution $v = \frac{y}{x}$, so $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Substituting into the equation from Step 1:

$$v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$$

$$x\frac{dv}{dx} = \sqrt{1+v^2}$$

We separate the variables v and x to integrate both sides:

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

Integrating both sides:

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

The integrals yield:

$$\ln\left|v + \sqrt{1+v^2}\right| = \ln|x| + \ln|C|$$

Where C is the constant of integration, expressed as $\ln|C|$ for simplification.

Using logarithm properties, $\ln(A) = \ln(B) + \ln(C) = \ln(BC)$:

$$\ln\left|v + \sqrt{1+v^2}\right| = \ln|Cx|$$

$$v + \sqrt{1+v^2} = Cx$$

Substitute back $v = \frac{y}{x}$ into the equation:

$$\frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2} = Cx$$

$$\frac{y}{x} + \sqrt{\frac{x^2+y^2}{x^2}} = Cx$$

$$\frac{y}{x} + \frac{\sqrt{x^2+y^2}}{|x|} = Cx$$

Assuming $x > 0$ for simplicity during this step, we multiply by x:

$$y + \sqrt{x^2+y^2} = Cx^2$$

(OR)

	Solve: $x\frac{dy}{dx} = y - x\sin\frac{y}{x}$. $\frac{dy}{dx} = \frac{y}{x} - \sin\left(\frac{y}{x}\right)$ Let $y = vx$, where v is a function of x. Differentiating both sides with respect to x gives: $$\frac{dy}{dx} = v + x\frac{dv}{dx}$$ Substitute $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ into the equation: $$v + x\frac{dv}{dx} = v - \sin(v)$$ Subtract v from both sides: $$x\frac{dv}{dx} = -\sin(v)$$ Rearrange to separate the variables v and x: $$\frac{dv}{\sin(v)} = -\frac{dx}{x}$$ $$\csc(v)\,dv = -\frac{dx}{x}$$ Integrate both sides of the equation: $$\int \csc(v)\,dv = -\int \frac{1}{x}\,dx$$ Using the standard integral $\int \csc(v)\,dv = \log\lvert\csc(v) - \cot(v)\rvert$: $$\log\lvert\csc(v) - \cot(v)\rvert = -\log\lvert x\rvert + \log(C)$$ $$\log\lvert\csc(v) - \cot(v)\rvert = \log\left\lvert\frac{C}{x}\right\rvert$$ $\csc\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right) = \frac{C}{x}$	
	Section–E **This section comprises of case based questions**	
Q.36	A rectangular visiting card is to contain 24 sq cm of printed matter. The margins at the top and bottom of the card are to be 1cm and the margins on the left and right are to be $1\frac{1}{2}$ cm. On the basis of given information, answer the following questions. (i) Write the expression for the area of the visiting card in terms of x. (ii) Obtain the dimensions of the card of minimum area.	(2) (2)

(i) $A(x) = 2x + 72x^{-1} + 30.$

(ii) 9 and 6

Q.37 A child cut a pizza with a knife. Pizza is circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edge of knife represents a straight line given by $x = \sqrt{3}\, y$.

Based on the above information, answer the following questions.

(i) Find the point of intersection of the edge of knife and pizza. (1)

(ii) Draw the graph of both circle and line and hence shade the smaller area bounded by edge of knife and pizza in the first quadrant. (1)

(iii) Using integration find the area of that smaller part. (2)

(i) **The points of intersection are $(\sqrt{3}, 1)$ and $(-\sqrt{3}, -1)$.**

(ii) Proper graph

(iii) $$\text{Total Area} = \frac{\sqrt{3}}{2} + \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \text{ sq. units.}$$

Q.38 There are two anti-aircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2. Both of them fired one shell at an airplane at the same time. (2) (2)

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

i)

This happens in two mutually exclusive ways:

1. Gun A hits AND Gun B misses: $P(A \cap B') = P(A) \times P(B') = 0.3 \times 0.8 = 0.24.$
2. Gun B hits AND Gun A misses: $P(B \cap A') = P(B) \times P(A') = 0.2 \times 0.7 = 0.14.$

$$P(\text{Exactly one hit}) = P(A \cap B') + P(B \cap A') = 0.24 + 0.14 = 0.38$$

We use the conditional probability formula: $P(B|E) = \frac{P(B \cap E)}{P(E)}$, where E is the event "exactly one hit".

Here, $B \cap E$ means "Gun B hit and exactly one hit occurred", which is the same as "Gun B hit and Gun A missed" ($B \cap A'$).

$P(B \cap A') = 0.14$ (from above).

$P(E) = 0.38$ (from above).

ii) $$P(B|\text{Exactly one hit}) = \frac{0.14}{0.38} = \frac{14}{38} = \frac{7}{19}$$

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